Unit 2: Dynamics

Lesson 1
Vectors and Forces

Dynamics: the study of the causes of motion. Why do objects move the way they do?

Force:

- A push or a pull (not necessarily resulting from physical contact).
- Measured in units of Newtons (N).
- 1 Newton = 1 kg·m/s^2.
- Vector quantity.

Free Body Diagrams: a diagram drawn to represent all the forces acting on an object at a particular time.

Forces can be balanced or unbalanced.

Consider:

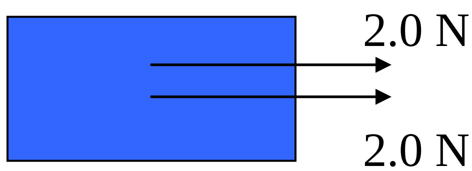
$$F_{net} = 0N$$

$$F_{net} = 6.0 - 3.0$$
$$F_{net} = 3.0\,N\,(E)$$

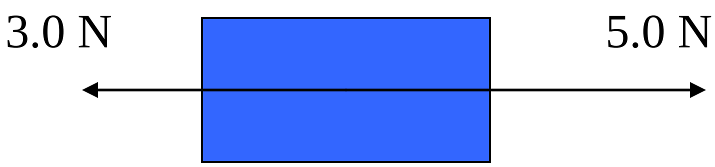
Equilibrant Force:
The force needed to balance the net force.

Example 1:
Find the net force acting on the objects shown.

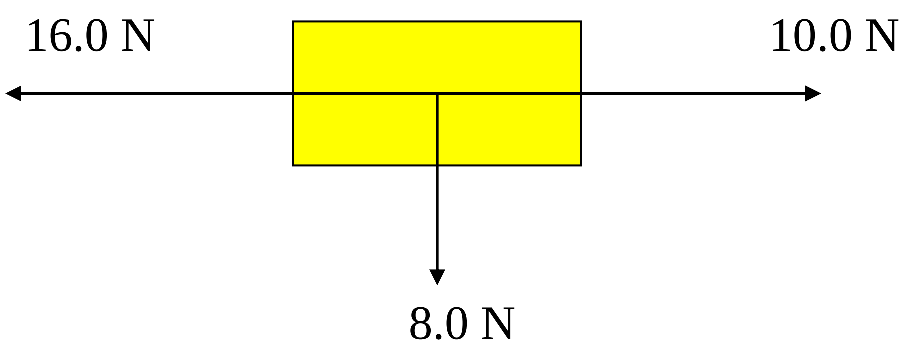
a)



b)

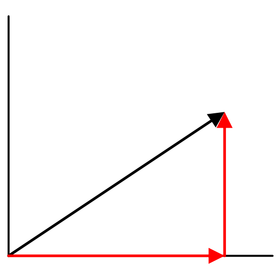


Example 2:
Find the net force acting on the object shown.



Force Components

Recall that any vector (including force) has both an *x* and a *y*-component.



$$\sin\theta = \frac{F_y}{F}$$

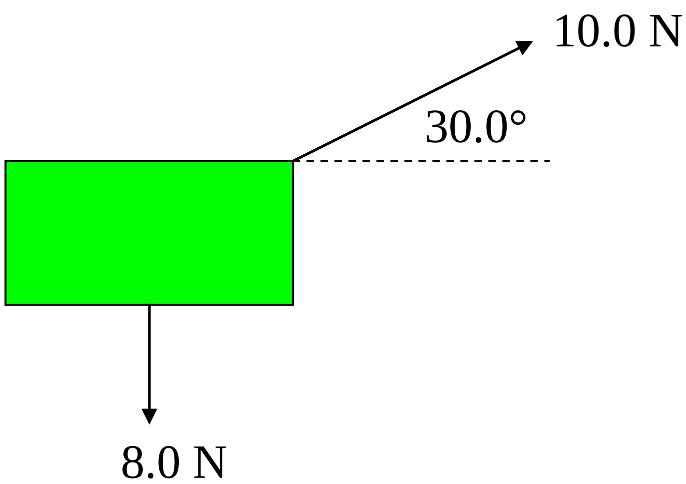
$$F_y = F\sin\theta$$

$$\cos\theta = \frac{F_x}{F}$$

$$F_x = F\cos\theta$$

Example 1:
A force of magnitude 125 N makes an angle of 30.0° with the horizontal. Determine its vertical and horizontal force components.

Example 2:
Find the net force acting on the object shown.



Example 3:
Find the net force for the following forces:

$$F_1 = 60.0N(50.0^\circ S\,of\,W)$$
$$F_2 = 80.0N(10.0^\circ S\,of\,E)$$
$$F_3 = 40.0N(70.0^\circ N\,of\,W)$$

Lesson 2

Newton's Laws:

- **Law of Inertia**
- **F=ma**
- **Action-Reaction**

Newton's First Law:
Objects at rest tend to stay at rest (and objects in constant motion will stay in motion) unless acted upon by an external unbalanced force.

Inertia: the property of an object to maintain its state of motion. The more massive an object, the more inertia it has.
**Note that if an object is at rest, the net force acting on it is 0. This is also true if an object is moving at a constant velocity. As soon as $F_{net} \neq 0$, the object will accelerate.

Inertial Frame of Reference:
One in which Newton's first law of motion is valid (i.e. any frame of reference that is not accelerating).
Consider a bus travelling in a straight line at a constant speed. The bus frame of reference is an inertial frame of reference. If you lay a ball at your feet it will stay there; if you push it, it will roll up the aisle. But if you lay the ball on the floor and the driver hits the brakes slightly, the ball will roll forward. To you, it looks like it is rolling forward for no reason – objects at rest should remain at rest. The frame of reference of the accelerating bus is no longer an inertial frame of reference.

Newton's Second Law:

Recall that if $F_{net} = 0$, an object will either move at a constant velocity or it will remain at rest.

If $F_{net} \neq 0$ the object is accelerating according to Newton's Second Law.

$$F_{net} = ma$$

where F_{net} is the net force in Newtons
m is the mass in kg
a is the acceleration in m/s^2

Example 1:
What is the acceleration of a 12 kg object that has a net force of 36 N (E)?

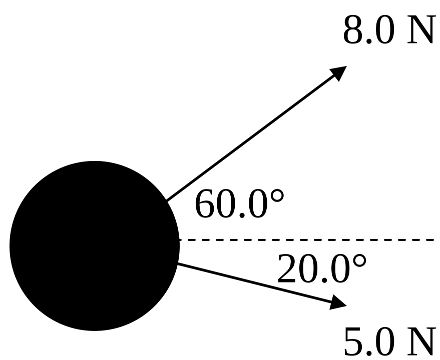
Example 2:
What is the mass of an object if an unbalanced force of 36 N gives it an acceleration of 3.0 m/s^2?

Example 3:
What force is necessary to accelerate a 1200 kg car along a horizontal surface from rest to 130 km/h in 8.0 s?

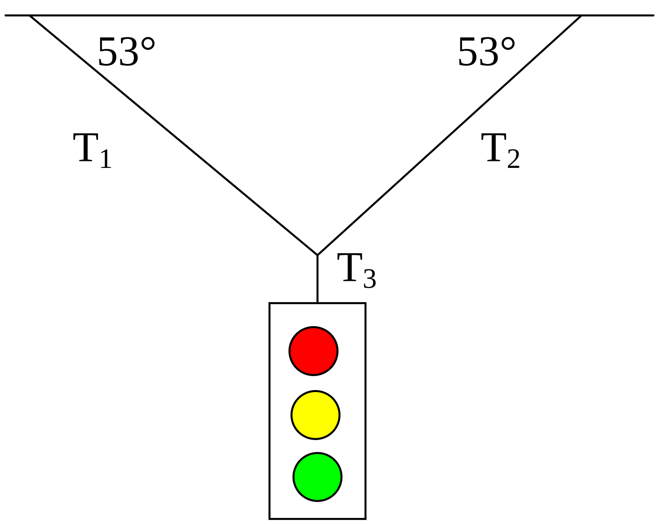
Example 4:
What mass would a sled on ice have if it requires a horizontal force of 100.0 N to change its velocity from 30.0 km/h to 120.0 km/h in 5.0 s?

Example 5:
What is the acceleration of a block having a mass of 0.50 kg that is being pulled in opposite directions by two children? Sean is pulling with a force of 3.0 N (W) and Diane is pulling 5.0 N (E). How far will it move in 3.0 s when these forces are exerted?

Example 6:
A hockey puck with a mass of 0.30 kg slides on the horizontal frictionless surface of an ice rink. Two forces act on the puck as shown. Determine the acceleration of the puck.



Example 7:
A traffic light weighing 100.0 N hangs from a cable tied to two other cables as shown. Find the tension in all three cables.



Example 8:

A person weighs a fish on a spring scale attached to the ceiling of an elevator.

a) elevator at rest
b) accelerating upward
c) accelerating downward
d) elevator cable breaks

Try:

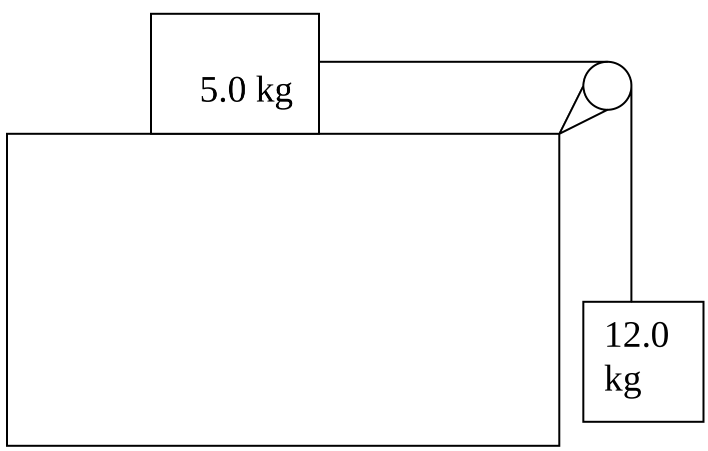
What would the tension be in a cable lifting an elevator and a person having a combined mass of 575 kg, moving at:

a) 5.0 m/s^2 (up)
b) 5.0 m/s^2 (down)

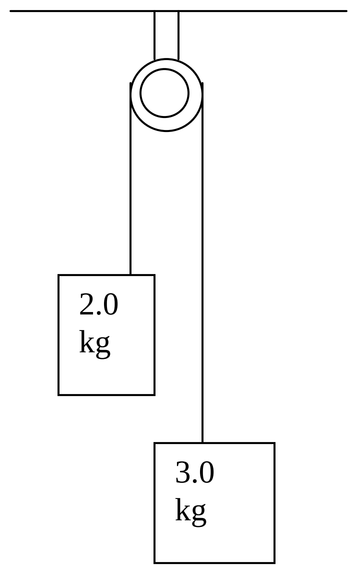
Example 9

For the blocks shown, calculate,

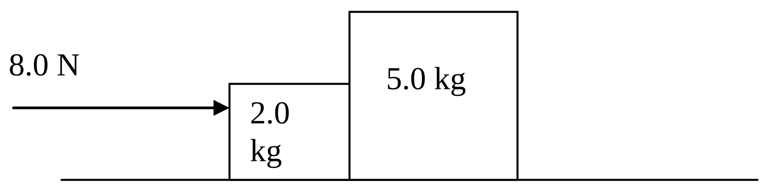
a) the acceleration of the blocks
b) the tension in the string



Example 10: Atwood's Machine
Determine the acceleration and the tension for the system of blocks shown below.



Example 11:
Calculate the acceleration of the system shown.



Newton's Third Law

If object A exerts a force on object B, then object B exerts an equal but opposite force on object A.

$$F_{AonB} = F_{BonA}$$

Consider holding a book in your hand.

If the hand exerts a 5 N force upward on the book, then the book exerts a 5 N force downward on the hand.

Note: Action-reaction forces always act on different objects.

Example:
Draw a free body diagram to show all the action-reaction pairs when a father pulls his daughter on a sled.

If these action-reaction pairs are opposite and equal, then how does the sled move forward?
Even though action-reaction pairs are equal, all forces in this situation are not equal. In this case the father can push harder on the ground than the ground can push on the sled.

Example:
If two people pull in opposite directions on the same object, is this an action-reaction pair?

No, these forces are acting on the same object and are not an action-reaction pair.

Lesson 3

Newton's Universal Law of Gravitation

Gravitational attraction exists between all objects in the universe. It is only noticed on a large scale because this force is very close to 0 when both objects are small.

$$F_G \; \alpha \; \frac{m_1 m_2}{r^2}$$

This is called an inverse square relationship (see p. 160).

If mass increases, then force increases.
If distance increases, then force decreases.

Example:
What is the effect on the gravitational force if one of the masses is doubled?

Example:
When two masses are a distance r apart, the gravitational force between them is 0.00010 N. What is the new force if one of the masses is tripled and the other is doubled?

Example:
By what factor will the gravitational force between two objects change if the distance between the masses decreases by a factor of 3?

Example:
If two masses each double while the distance between them also doubles, by what factor will the gravitational force change?

To convert the proportionality into an equation, we introduce a proportionality constant G.

G = universal gravitational constant = $6.67 \times 10^{-11} N \cdot \frac{m^2}{kg^2}$

So,

$$F_G = \frac{G m_1 m_2}{r^2}$$

where,
G = universal gravitational constant
m_1 and m_2 = masses (kg)
r = distance between masses (m)

Example:
What happens to your weight if you go down a mine shaft towards the centre of Earth?

Example:
What is the force of gravity between a girl of mass 49 kg and her cat of mass 5.1 kg if they are separated by a distance of 1.2 m?

Example:
Calculate the force of attraction between a 1.0 kg watermelon and Earth of mass 5.98×10^{24} kg if the object is sitting on Earth's surface.

Note that $g = 9.80$ m/s^2 and as long as the object is at or near the surface Earth, we use the equation $F = mg$ to find the weight of an object.

Example:
If your mass is 100.0 kg, what will be the gravitational force exerted on you by Earth if you are in a space ship that 12800 km above Earth's surface? (The radius of Earth is 6400 km).

Lesson 4

Momentum

Momentum = mass × velocity

- Vector quantity
- Units of kg·m/s
- Direction of momentum is same as direction of velocity.

Example:
What is the momentum of a 2.0×10^3 kg car that has a velocity of 12.8 m/s (E)?

Example:
What is the velocity of a 50.0 g bullet that has a momentum of 24.75 kg·m/s (N)?

Impulse
Which is heavier, a 15 N feather pillow or a 15 N brick?
Which would you rather be hit on the head with? Why?
Consider how long it takes for each to stop.

Brick ~ 0.010 s
Pillow ~ 0.50 s (fifty times longer than brick)

Consider two other factors:

- The force applied to your head
- The time over which the force is applied

Impulse = force × time

$J = F_{net}\Delta t$ (units of N·s)

But

$$F_{net} = ma$$

and

$$a = \frac{\Delta v}{\Delta t}$$

and

$$p = mv$$

Thus

$$F_{net}\Delta t = ma\Delta t$$

$$F_{net}\Delta t = m\frac{\Delta v}{\Delta t}\Delta t$$

$$F_{net}\Delta t = m\Delta v$$

$$J = F_{net}\Delta t = \Delta p$$

So impulse = change in momentum.
The impulse of a force is equal to the change in momentum of the object that is experiencing the impulse.

So if the impulse for the pillow and the brick is the same, we can see that a longer time to stop results in a smaller force to the head.

Example:
A 100.0 g golf ball leaves the tee at 100.0 m/s. The club is in contact with the ball for 0.040 s. Determine the force exerted on the ball by the club.

Example:
A 180 g baseball travelling at 30.0 m/s (E) is struck by a bat. It then travels at 35.0 m/s (W).
a) What is the impulse given to the ball by the bat?
b) How long is the bat in contact with the ball if it exerts a force of 8.0×10^3 N (W) on the ball?

Example:
Explain why it is easier to drive a nail with a steel hammer than with a rubber mallet.

The steel hammer has less give (i.e shorter stopping time) and thus for the same change in momentum will give a greater net force

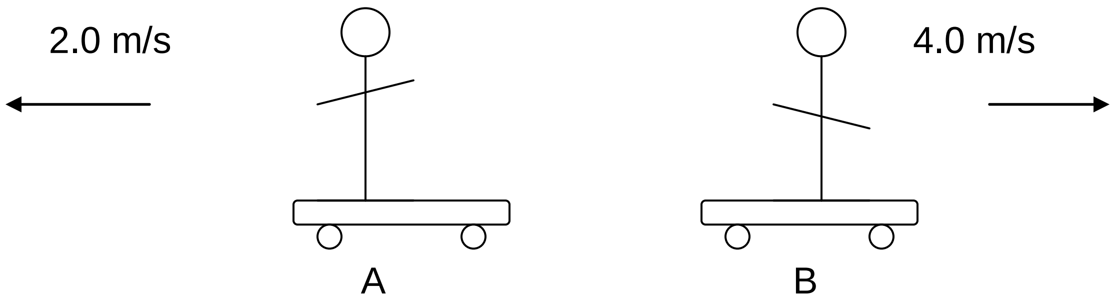
Example:
Explain why when you jump from a height, you should always bend your knees as you land.

Note that the area under a graph of force versus time will give the impulse (see p. 283-287).

Conservation of Momentum

Recall: p=mv

Consider the situation below:



Show that momentum is conserved in this situation.

Example:
Cart A has a mass of 2.0 kg and is travelling at 10.0 m/s to the right. Cart B has a mass of 3.0 kg and is moving to the left at 4.0 m/s. The carts collide and stick together. With what velocity will the carts move after they stick together?

Try:
Cart A has a mass of 2.0 kg and is travelling at 10.0 m/s to the right. Cart B has a mass of 3.0 kg and is moving to the left at 6.0 m/s. Cart B collides with cart A which causes cart A to rebound at 4.0 m/s. What is the velocity of cart B after the impact? (Ans: 3.3 m/s)

The law of conservation of momentum is typically stated as:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

There are several possible scenarios:

- Objects bounce off one another
- Objects both move in the same direction
- One object stops
- Objects stick together

Example:
A 50.0 g bullet is shot from a 4.0 kg rifle. The bullet leaves the rifle at 400.0 m/s. What is the recoil velocity of the rifle?